6.4: Working with Power Series

Def: A _____ is an equation relating some functions and their _____

Here are some examples:

Note: The equals sign in a differential equation means...

Q. What is a solution to a differential equation?

Ex 1 Show that $y = e^x + x^3$ is a solution to $\frac{dy}{dx} - 3x^2 = y - x^3$

Ex 2 Is $y = 2e^x + x^3$ a solution to $\frac{dy}{dx} - 3x^2 = y - x^3$? What about $y = e^x + 2x^3$?

Ex 3 Solve $\frac{dy}{dx} = y$

Ex 4 Use power series to solve $\frac{dy}{dx} = y$.

Ex 5 Consider the differential equation $\dfrac{d^2y}{dx^2} + y = 0$.

(a) Solve the differential equation.

(b) Suppose you know that your solution satisfies $\frac{dy}{dx}\big|_{x=0} = -7$ and $y(0) = 2$. Find an exact formula for y as a function of x.

Ex 6 Find the 5th Maclaurin polynomial for the power series solution to $\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$ where $y(0) = 1$ and $\frac{dy}{dx}\big|_{x=0} = 3$.